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DETERMINATION OF THE g-VALUES OF THE TERMS OF A SPECTRUM

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It is difficult to obtain the g factors of all the terms of a spectrum, owing to the fact that in the Zeeman-effect a total resolution is obtained only for a small number of lines. Nevertheless it is of the greatest importance to obtain the g of all the terms.

When the spectrum is structurally analysed, some more lines can be resolved, if the g of one of the terms is known, as was shown by Shenstone and Blair [1]), Russell [2]), Martinez Sancho [3]), and Catalán and Poggio [4]).

To obtain the values of g in these cases the solution of simple equations is required, but the computation becomes very elaborate, when the lines are numerous. As the coefficients in the equations are always the same, we have calculated them once for all, and we have combined them in tables, which enables us to obtain the values of g easily. As the method here adopted might be useful for others, we have decided to make it known.

We shall call the g values of the two levels which form the line g_x and g_y, g_y being the g corresponding with the level of the greatest J.

ODD MULTIPLICITIES

When the value of the g of one of the terms is known, we calculate the g of the other term.

Example: WI [4]) 3654,20. $a\,^5D_2 — 36^0_1$; (0) 1,56.

1) Shenstone a Blair. Phil. Mag. **8**, 765, 1929.
2) Russell. Phys. Rev. **36**, 1590, 1930.
3) Martinez Sancho. An. Soc. Esp. Fis. Quim. **30**, 867, 1932.
4) Catalán y Poggio. An. Soc. Esp. Fis. Quim. **32**, 255, 1934.

The value of g_y derived from the investigation of other lines is:

$$g_y = 1{,}50$$

Applying the corresponding formula we obtain

$$3.00 \times 1.50 \pm 2.00 \times 1.56 = 1.38$$

value very close to others of the level 36^0_1.

Case $J_x \neq J_y$			
J_x	J_y	g_x	g_y
0	1	0	$1.00\ B\sigma$
1	2	$3.00\ g_y \pm 2.00\ B\sigma$	$0.33\ g_x \mp 0.67\ B\sigma$
2	3	$2.00\ g_y \pm 1.00\ B\sigma$	$0.50\ g_x \mp 0.50\ B\sigma$
3	4	$1.67\ g_y \pm 0.67\ B\sigma$	$0.60\ g_x \mp 0.40\ B\sigma$
4	5	$1.50\ g_y \pm 0.50\ B\sigma$	$0.67\ g_x \mp 0.33\ B\sigma$
5	6	$1.40\ g_y \pm 0.40\ B\sigma$	$0.71\ g_x \mp 0.29\ B\sigma$
6	7	$1.33\ g_y \pm 0.33\ B\sigma$	$0.75\ g_x \mp 0.25\ B\sigma$

Example: WI 3526.86 $3_3 - 83^0_3$ (0.82) 1.09

$$g_x = 1.09 + 0.194 \times 0.82 = 1.09 + 0.16 = 1.25$$

$$g_y = B_\sigma - 0.194 \times 0.82 = 1.09 - 0.16 = 0.93.$$

The value 1,25 corresponds to the level 83^0_3, since the value found by studying other lines of the same level is 1,24.

The value 0,93 corresponds to the level 3_3. From the study of other lines 0,92 has been found.

Case $J_x = J_y$		
$J_x = J_y$	g_x	g_y
1	$B\sigma + 0.500\ B\pi$	$B\sigma - 0.500\ B\pi$
2	$B\sigma + 0.278\ B\pi$	$B\sigma - 0.278\ B\pi$
3	$B\sigma + 0.194\ B\pi$	$B\sigma - 0.194\ B\pi$
4	$B\sigma + 0.150\ B\pi$	$B\sigma - 0.150\ B\pi$
5	$B\sigma + 0.122\ B\pi$	$B\sigma - 0.122\ B\pi$
6	$B\sigma + 0.103\ B\pi$	$B\sigma - 0.103\ B\pi$
7	$B\sigma + 0.090\ B\pi$	$B\sigma - 0.090\ B\pi$

To obtain the Zeeman-effect when the values of the two g's are known

Example: WI. 3468,41 $4_2 — 86^0_3$.

The parallel component will be 0, the vertical one will be calculated by the formula:

$$B_\sigma = 2{,}00\, g_y — 1{,}00\, g_x.$$

The values derived from the study of other lines being:

$$4_2,\ g_x = 1{,}10 \text{ and for } 86^0_3,\quad g_y = 1{,}18, \text{ we get:}$$

$$B_\sigma = 2{,}00 \times 1{,}18 — 1{,}00 \times 1{,}10 = 1{,}26$$

The calculated effect is (0) 1,26, the observed effect is also (0)1,26.

Case $Jx \neq Jy$			
Jx	Jy	$B\pi$	$B\sigma$
0	1	Zero values in all cases	gy
1	2		1.50 gy—0.50 gx
2	3		2.00 gy—1.00 gx
3	4		2.50 gy—1.50 gx
4	5		3.00 gy—2.00 gx
5	6		3.50 gy—2.50 gx
6	7		4.00 gy—3.00 gx

Example: WI. 4109,76 $3_3 — 60^0_3$.

The values assumed for g are:

$$\text{for } 3_3,\ g = 0{,}92 \text{ and for } 60^0_3,\ g = 1{,}12$$

$$B_\pi = 2{,}57\,(g_x — g_y) = 2{,}57 \times 0{,}20 = 0{,}51$$

$$B_\sigma = 0{,}50\,(g_x + g_y) = 0{,}50 \times 2{,}04 = 1{,}02.$$

The calculated effect is (0,51) 1,02 and the measured effect is (0,56) 1,02.

Case $Jx = Jy$		
$Jx = Jy$	$B\pi$	$B\sigma$
1	1.00 (gx — gy)	In all cases 0.50 (gx — gy)
2	1.79 (gx — gy)	
3	2.57 (gx — gy)	
4	3.33 (gx — gy)	
5	4.09 (gx — gy)	
6	4.85 (gx — gy)	
7	5.55 (gx — gy)	

EVEN MULTIPLICITIES

To calculate the g of the other term when the value of one g is known.

Example: Cr. II [1]) $\lambda = 2678{,}802$; $a\,^6D_{1\frac{1}{2}} - z\,^6D_{2\frac{1}{2}}$; (0) 1,61.

For the level $z\,^6D_{2\frac{1}{2}}$ (that of the greatest J) the value of $g = 1{,}79$ is known.

Applying the formula of series $1\frac{1}{2}$, $2\frac{1}{2}$, we have

$$g_x = 2{,}333\ g_y - 1{,}333\ B_\sigma = 2{,}333 \times 1{,}79 - 1{,}333 \times 1{,}61 =$$
$$= 4{,}17 - 2{,}14 = 2{,}03.$$

This value is very close to that derived from other lines: 2,01.

Case $J_x \neq J_y$			
J_x	J_y	g_x	g_y
$\frac{1}{2}$	$1\frac{1}{2}$	$5\ g_y - 4\ B_\sigma$	$0.800\ B_\sigma + 0.200\ g_x$
$1\frac{1}{2}$	$2\frac{1}{2}$	$2.333\ g_y - 1.333\ B_\sigma$	$0.571\ B_\sigma + 0.488\ g_x$
$2\frac{1}{2}$	$3\frac{1}{2}$	$1.800\ g_y - 0.800\ B_\sigma$	$0.444\ B_\sigma + 0.555\ g_x$
$3\frac{1}{2}$	$4\frac{1}{2}$	$1.571\ g_y - 0.571\ B_\sigma$	$0.363\ B_\sigma + 0.636\ g_x$
$4\frac{1}{2}$	$5\frac{1}{2}$	$1.444\ g_y - 0.444\ B_\sigma$	$0.307\ B_\sigma + 0.692\ g_x$
$5\frac{1}{2}$	$6\frac{1}{2}$	$1.363\ g_y - 0.363\ B_\sigma$	$0.266\ B_\sigma + 0.733\ g_x$

Example: Cr. II $\lambda = 3147{,}224$; $a\,^4D_{3\frac{1}{2}} - z\,^4F_{3\frac{1}{2}}$; (0,63) 1,46.
The formulae that are to be applied are :

$$g_y = B_\sigma - 0{,}169\ B_\pi \text{ y } g_x = B_\sigma + 0.169\ B_\pi$$
$$g_y = 1.46 - 0.169 \times 0.63 = 1.46 - 0.11 = 1.35$$
$$g_x = 1.46 + 0.169 \times 0.63 = 1.46 + 0.11 = 1.57.$$

The values derived from other lines are:

$$g_y = 1{,}25$$
$$g_x = 1{,}54.$$

Case $J_x = J_y$		
$J_x = J_y$	g_y	g_x
$\frac{1}{2}$	$B_\sigma - B_\pi$	$B_\sigma + B_\pi$
$1\frac{1}{2}$	$B_\sigma - 0.357\ B_\pi$	$B_\sigma + 0.357\ B_\pi$
$2\frac{1}{2}$	$B_\sigma - 0.228\ B_\pi$	$B_\sigma + 0.228\ B_\pi$
$3\frac{1}{2}$	$B_\sigma - 0.169\ B_\pi$	$B_\sigma + 0.169\ B_\pi$
$4\frac{1}{2}$	$B_\sigma - 0.134\ B_\pi$	$B_\sigma + 0.134\ B_\pi$
$5\frac{1}{2}$	$B_\sigma - 0.111\ B_\pi$	$B_\sigma + 0.111\ B_\pi$
$6\frac{1}{2}$	$B_\sigma - 0.095\ B_\pi$	$B_\sigma + 0.095\ B_\pi$

1) M. A. Catalán. An. Soc. Esp. **28**, 611, 1930 y Handbuch d. Spec. Kayser u Konen (**8**, I, p. 573, 1932).

To obtain the Zeemaneffect, when the values of the two g's are known

Example: Cr II. λ 3754,60; $b\,^4D_{1\frac{1}{2}} — z\,^4F_{2\frac{1}{2}}$; (0) 0,89.

The g-values obtained by the study of other lines are:

$$\text{for } b^4D_{1\frac{1}{2}},\ g = 1{,}24 \text{ and for } z^4F_{2\frac{1}{2}},\ g = 1{,}04.$$

$B_\sigma = 1{,}75\ g_y — 0{,}75\ g_x$, g_y representing as usual the g for the highest value of J. Hence:

$$B_\sigma = 1{,}75 \times 1{,}04 — 0{,}75 \times 1{,}24 = 0{,}89.$$

The effect measured is also 0,89.

Case $Jx \neq Jy$			
Jx	Jy	$B\pi$	$B\sigma$
$\frac{1}{2}$	$1\frac{1}{2}$	Zero values in all cases	$1.25\ gy — 0.25\ gx$
$1\frac{1}{2}$	$2\frac{1}{2}$		$1.75\ gy — 0.75\ gx$
$2\frac{1}{2}$	$3\frac{1}{2}$		$2.25\ gy — 1.25\ gx$
$3\frac{1}{2}$	$4\frac{1}{2}$		$2.75\ gy — 1.75\ gx$
$4\frac{1}{2}$	$5\frac{1}{2}$		$3.25\ gy — 2.25\ gx$
$5\frac{1}{2}$	$6\frac{1}{2}$		$3.75\ gy — 2.75\ gx$

Example: Cr. II $\lambda = 4132{,}45$; $a\,^4P_{1\frac{1}{2}} — z\,^4D_{1\frac{1}{2}}$; (0,81) 1,48.

The g-values obtained from other lines are:

$$a\,^4P_{1\frac{1}{2}},\ g = 1{,}76 \text{ and}$$
$$z\,^4D_{1\frac{1}{2}},\ g = 1{,}30.$$

We get for B_σ:

$$B_\sigma = 0{,}50\ (1{,}76 + 1{,}30) = 1{,}53 \text{ and for } B_\pi:$$
$$B_\pi = 1{,}40\ (1{,}76 — 1{,}30) = 0{,}64.$$

Case $Jx = Jy$		
$Jx = Jy$	$B\pi$	$B\sigma$
$\frac{1}{2}$	$0.500\ (gx—gy)$	
$1\frac{1}{2}$	$1.400\ (gx—gy)$	
$2\frac{1}{2}$	$2.185\ (gx—gy)$	
$3\frac{1}{2}$	$2.952\ (gx—gy)$	$0.50\ (gx + gy)$
$4\frac{1}{2}$	$3.712\ (gx—gy)$	
$5\frac{1}{2}$	$4.468\ (gx—gy)$	
$6\frac{1}{2}$	$5.222\ (gx—gy)$	

The practical method which we apply for the calculation of the values of g is the following:

A table is prepared, in the vertical columns of which the even levels are recorded, and in the horizontal rows the odd levels. The two values of g referring to every line are placed in the corresponding place.

We start calculating the values of g derived from the completely resolved lines and from the unresolved lines which directly yield the *two* values of g, e.g. on our partial table of WI, lines as $a\,^5D_0 — 14^0_1$ and $a^7\,s_3 — 35^3_0$ respectively. Afterwards these values of g enable us to calculate the unresolved lines because when the value of g of a level is known the g of the other level can be calculated by means of the preceding formulae. Finally the values ensuing from the unresolved lines $\Delta J = 0$ are considered.

In the following table the values in italics correspond to well resolved lines; *m* indicates that the mean value has been taken to calculate the other g. The remaining values with no special sign derive from unresolved lines which give the values of g directly.

In the righthand column and in the bottom row the mean values of g are recorded.

Example of calculation of the g-values in WI [1])

	a^5D_0	a^5D_1	a^5D_2	a^5D_3	a^5D_4	a^7S_3	Accepted g-value
14°_1	**0.00** **2.53**	**1.43** **2.40**	**1.50** **2.54**				2.53 A
17°_2			**1.47** **1.88**	**1.59** **2.02**		m 1.99 1.93	1.93 A
20°_1	**0.00** **0.50**	**1.58** **0.56**					0.51 A
21°_3			m 1.49 2.05	**1.49** **1.77**	1.48 1.86 m	**2.00** **1.76**	1.77 A
22°_2		m 1.49 1.94	1.51 1.51			m 1.99 1.89	1.91 D
24°_3			m 1.49 1.70	1.45 1.71	1.56 1.72 m	**2.00** **1.75**	1.75 A
32°_2		m 1.49 1.25	1.47 1.25			1.91 **1.22**	1.23 A
35°_3			m 1.49 1.67	1.61 1.61	1.46 1.62 m	2.00 1.60	1.61 C
43°_3			m 1.49 1.43	1.43 1.43		2.12 1.52	1.44 C
55°_2		m 1.49 1.52	1.51 1.51			1.98 1.52	1.52 B
	0.00 A	1.49 A	1.49 A	1.49 A	1.49 A	2.00 A	Accepted g-value

A. g-values determined from different lines well resolved.
B. " " " one line " "
C. " " " different lines not resolved, giving good agreement.
D. " " " " " " " giving not good agreement.

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1) See Catalán y Poggio, loc. cit.